A New and moſt Accurate

THEORY OF THE Moon's Motion;

Whereby all her Irregularities may be ſolved, and her Place truly calculated to Two Minutes.

Written by
That Incomparable Mathematician
Mr. ISAAC NEWTON,
K

And Publiſhed in *Latin* by
Mr. DAVID GREGORY
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TO THE

READER.

THE Irregularity of the Moon's Motion hath been all along the juſt Complaint of Aſtronomers; and indeed I have always look'd upon it as a great Misfortune that a Planet ſo near us as the Moon is, and which might be ſo wonderfully uſe-

ful to us by her Motion, *as well as her* Light *and* Attraction *(by which our Tides are chiefly occaſioned) ſhould have her Orbit ſo unaccountably various, that it is in a manner vain to depend on any Calculation of an* Eclipſe, *a* Tranſit, *or an* Appulſe *of her, tho never ſo accurately made. Whereas could her Place be but truly calculated, the Longitudes of Places would be found every where at Land with great Facility, and might be nearly gueſs'd at Sea without the help of a Teleſcope, which cannot there be uſed.*

This Irregularity of the Moon's Motion depends (as is now well known, ſince Mr. Newton *hath demonſtrated the Law of Univerſal Gravitation) on the Attraction of* *of the Sun, which perturbs the Motion of the Moon (and of all other* Satellites *or ſecondary Planets) and makes her move ſometimes faſter and ſometimes ſlower in her Orbit; and makes conſequently an Alteration in the Figure of that Orbit, as well as of its Inclination to the Plain of the Ecliptick. But this being* now *to be accounted for, and reduced to a Rule; by this Theory ſuch Allowances are made for it, as that the Place of the Planet ſhall be truly* Equated.

This therefore being perfectly New, *and what the Lovers of Aſtronomy have a long while been put in hopes to receive from the* Great Hand *that hath now finiſhed*

ſhed it; I thought it would be of good ſervice to our Nation to give it an Engliſh *Dreſs, and publiſh it by it ſelf: For as Dr.* Gregory's Aſtronomy *is a large and ſcarce Book, it is neither every ones Money that can purchaſe it, nor Acquaintance that can procure it; and beſides I hope we have a great many Perſons in* England *that have* Skill *and* Patience *enough to calculate a Planet's Place, who yet it may be don't well enough underſtand the* Latin *Tongue to make themſelves Maſters of this Theory in the Author's own Words. At leaſt I perſwade my ſelf, that a Theory ſo eaſy and plain as this, which carries along with it ſuch a Pretence to Exactneſs, will encourage many Perſons to imploy themſelves in Aſtronomical*

nomical Calculation, which before poſſibly they neglected, becauſe they judged there was but little Exactneſs to be attained in it. And this would be a very uſeful way of ſpending their leiſure Hours; and if they would oblige us with the Publication of good Ephemerides, Tables, *&c. they would ſoon enflame others with a Deſire of purſuing theſe kind of Studies.*

The Famous Mr. *Iſaac Newton*'s THEORY OF THE MOON.

THIS Theory hath been long expected by the Lovers of Art, and is now publiſh'd in Dr. *Gregory*'s Aſtronomy, in Mr. *Newton*'s own Words.

By this Theory, what by all Aſtronomers was thought moſt difficult and almoſt impoſſible to be done, the Excellent Mr. *Newton* hath now effected, *viz.* to determine the Moon's Place even in her Quadratures, and all other Parts of her Orbit, beſides the Syzygys, ſo accurately by Calculation, that the Difference between that and her true Place in the Heavens ſhall ſcarce be two Minutes, and is uſually ſo ſmall, that it may well enough be reckon'd only as a Defect in the Obſervation. And this Mr. *Newton* experienced by comparing it with very many Places of the Moon obſerv'd by Mr. *Flamſtead*, and communicated to him.

The Royal Obſervatory at *Greenwich* is to the Weſt of the Meridian of *Paris* 2°. 19′. Of *Uraniburgh* 12°. 51′. 30″. And of *Gedanum* 18°. 48′.

The mean Motions of the Sun and Moon, accounted from the Vernal Equinox at the Meridian of *Greenwich*, I make to be as followeth. The laſt Day of *December* 1680. at Noon (Old Stile) the mean Motion of the Sun was $9^{\text{Sign.}}\ 20^{\circ}.\ 34'.\ 46''.$ Of the Sun's Apogæum was $3^{\text{Sign.}}\ 7^{\circ}.\ 23'.\ 30''.$

The mean Motion of the Moon at that time was $6^{\text{Sign.}}\ 1^{\circ}.\ 35'.\ 45''.$ And of her Apogee $8^{\text{Sign.}}\ 4^{\circ}.\ 28'.\ 5''.$ Of the Aſcending Node of the Moon's Orbit $5^{\text{Sign.}}\ 24^{\circ}.\ 14'.\ 35''.$

And on the laſt Day of *December* 1700. at Noon, the mean Motion of the Sun was $9^{\text{Sign.}}\ 20^{\circ}.\ 43'.\ 50''.$ Of the Sun's Apogee $3^{\text{Sign.}}\ 7^{\circ}.\ 44'.\ 30''.$ The mean Motion of the Moon was $10^{\text{Sign.}}\ 15^{\circ}.\ 19'.\ 50''.$ Of the Moon's Apogee $11^{\text{Sign.}}\ 8^{\circ}.\ 18'.\ 20''.$ And of her aſcending Node $4^{\text{Sign.}}\ 27^{\circ}.\ 24'.\ 20''.$ For in 20 *Julian* Years or 7305 Days, the Sun's Motion is 20 Revolut. 0 Sign. 0°. 9′. 4″. And the Motion of the Sun's Apogee 21′. 0″.

The Motion of the Moon in the ſame Time is 247 Rev. 4 Sign. 13°. 34′. 5″. And the Motion of the Lunar Apogee is 2 Revol. 3 Sign. 3°. 50′. 15″. And the Motion of her Node 1 Revol. 0 Sign. 26°. 50′. 15″.

All which Motions are accounted from the Vernal Equinox: Wherefore if from them there be ſubtracted the Receſſion or Motion of the Equinoctial Point *in Antecedentia* during that ſpace, which is 16′. 0″. there will remain the Motions in reference to the Fixt Stars in 20 *Julian* Years; *viz.* the Sun's 19 Revol. 11 Sign. 29°. 52′. 24″. Of his Apogee 4′. 20″. And the Moon's 247 Revol. 4 Sign. 13°. 17′. 25″. Of her Apogee 2 Revol. 3 Sign. 3°. 33′. 35″. And of the Node of the Moon 1 Revol. 0 Sign. 27°. 6′. 55″.

According to this Computation the *Tropical Year* is $365^{\text{Days.}}$ $5^{\text{Hours.}}$ $48'$. $57''$. And the *Sydereal Year* is $365^{\text{Days.}}$ $6^{\text{Hours.}}$ $9'$. $14''$.

Theſe mean Motions of the Luminaries are affected with various Inequalities: Of which,

1. There are the Annual Equations of the aforeſaid mean Motions of the Sun and Moon, and of the Apogee and Node of the Moon.

The Annual Equation of the mean Motion of the Sun depends on the Eccentricity of the Earth's Orbit round the Sun, which is $16\frac{11}{12}$ of ſuch Parts, as that the Earth's mean Diſtance from the Sun ſhall be 1000: Whence 'tis called the *Equation of the Centre*; and is when greateſt 1°. $56'$. $20''$.

The greateſt Annual Equation of the Moon's mean Motion is $11'$. $49''$. of her Apogee $20'$. and of her Node $9'$. $30''$.

And theſe four Annual Equations are always mutually proportional one to another: Wherefore when any of them is at the greateſt, the other three will alſo be greateſt; and when any one leſſens, the other three will alſo be diminiſhed in the ſame *Ratio*.

The Annual Equation of the Sun's Centre being given, the three other correſponding Annual Equations will be alſo given; and therefore a Table of *that* will ſerve for all. For if the Annual Equation of the Sun's Centre be taken from thence, for any Time, and be called P, and let $\frac{1}{10}P = Q$, $Q + \frac{1}{60}Q = R$, $\frac{1}{6}P = D$, $D + \frac{1}{30}D = E$, and $D - \frac{1}{60}D = 2F$; then ſhall the Annual Equation of the Moon's mean Motion for that time be R, that of the Apogee of the Moon will be E, and that of the Node F.

Only obſerve here, that if the Equation of the Sun's Centre be required to be *added*; then the Equation of the Moon's mean Motion muſt be *ſubtracted*, that of her Apogee muſt be *added*, and that of the Node *ſubducted*. And on the contrary, if the Equation of the Sun's Centre were to be *ſubducted*, the Moon's Equation muſt be *added*, the Equation of her Apogee *ſubducted*, and that of her Node *added*.

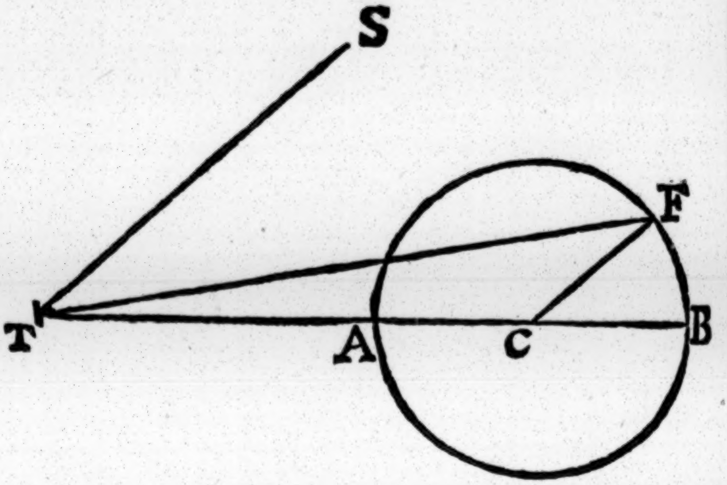
There is alſo an *Equation of the Moon's mean Motion* depending on the Situation of her Apogee in reſpect of the Sun; which is *greateſt* when the Moon's Apogee is in an Octant with the Sun, and is nothing at all when it is in the Quadratures or Syzygys. This Equation, when greateſt, and the Sun *in Perigæo*, is 3′. 56″. But if the Sun be *in Apogæo*, it will never be above 3′. 34″. At other Diſtances of the Sun from the Earth, this Equation, when greateſt, is reciprocally as the Cube of ſuch Diſtance. But when the Moon's Apogee is any where but in the *Octants*, this Equation grows leſs, and is moſtly at the ſame diſtance between the Earth and Sun, as the Sine of the double Diſtance of the Moon's Apogee from the next Quadrature or Syzygy, to the Radius.

This is to be *added* to the Moon's Motion, while her Apogee paſſes from a Quadrature with the Sun to a Syzygy; but is to be *ſubtracted* from it, while the Apogee moves from the Syzygy to the Quadrature.

There is moreover another *Equation of the Moon's Motion*, which depends on the Aſpect of the Nodes of the Moon's Orbit with the Sun: and this is *greateſt* when her Nodes are in *Octants* to the Sun, and vaniſhes quite, when they come to their Quadratures or Syzygys. This Equation is proportional to the Sine of the double Diſtance of the

Node from the next Syzygy or Quadrature; and at greateſt is but 47″. This muſt be *added* to the Moon's mean Motion, while the Nodes are paſſing from their Syzygys with the Sun to their Quadratures with him; but *ſubtracted* while they paſs from the Quadratures to the Syzygys.

From the Sun's true Place take the equated mean Motion of the Lunar Apogee, as was above ſhewed, the Remainder will be the Annual Argument of the ſaid Apogee. From whence the *Eccentricity of the Moon.* and the *ſecond Equation* of her Apogee may be compar'd after the manner following *(which takes place alſo in the Computation of any other intermediate Equations.)*



Let T repreſent the Earth, T S a Right Line joining the Earth and Sun, T A C B a Right Line drawn from the Earth to the middle or mean Place of the Moon's Apogee, equated, as above: Let the Angle S T A be the Annual Argument of the aforeſaid Apogee, T A the leaſt Eccentricity of the Moon's Orbit, T B the greateſt. Biſſect A B in C; and on the Centre C with the Diſtance A C deſcribe a Circle A F B, and make the Angle B C F = to the double of the Annual Argument. Draw the Right Line T F, that ſhall be the Eccentricity of the Moon's Orbit; and the Angle B T F is the ſecond Equation of the Moon's Apogee required.

In order to whoſe Determination let the mean Diſtance of the Earth from the Moon, or the Semidiameter of the Moon's Orbit, be 1000000; then ſhall its *greateſt Eccentricity* T B be 66782 ſuch Parts; and the *leaſt* T A, 43319. So that the greateſt Equation of the Orbit, *viz.* when the Apogee is in the Syzygys, will be 7°. 39′. 30″. or perhaps 7°. 40′. (for I ſuſpect there will be ſome Alteration according to the Poſition of the Apogee in ♋ or in ♑.) But when it is in Quadrature to the Sun, the greateſt Equation aforeſaid will be 4°. 57′. 56″. and the greateſt Equation of the Apogee 12°. 15′. 4″.

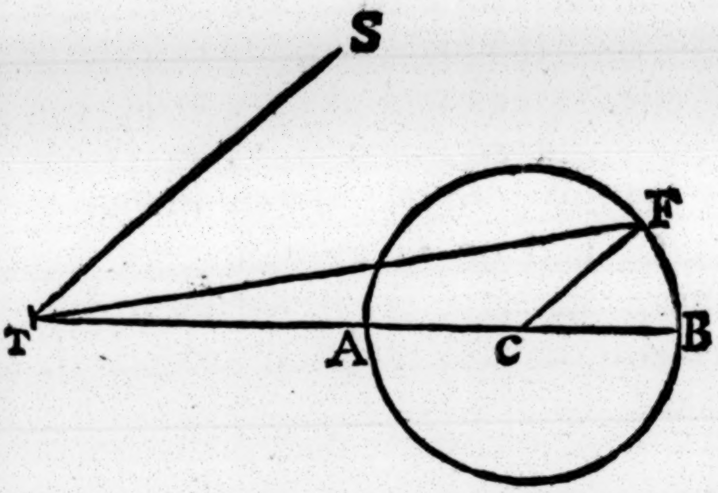
Having from theſe Principles made a Table of the Equation of the Moon's Apogee, and of the Eccentricitys of her Orbit to each degree of the Annual Argument, from whence the Eccentricity T F, and the Angle B T F (*viz.* the ſecond and principal Equation of the Apogee) may eaſily be had for any Time required; let the Equation thus found be added, to the firſt Equated Place of the Moon's Apogee, if the Annual Argument be leſs than 90°, or greatear than 180°, and leſs than 270; otherwiſe it muſt be ſubducted from it: and the Sum or Difference ſhall be the Place of the Lunar Apogee ſecondarily equated; which being taken from the Moon's Place equated a third time, ſhall leave the mean Anomaly of the Moon correſponding to any given Time. Moreover, from this mean Anomaly of the Moon, and the before-found Eccentricity of her Orbit, may be found (by means of a Table of Equations of the Moon's Centre made to every degree of the mean Anomaly, and ſome Eccentricitys, *viz.* 45000, 50000, 55000, 60000, and 65000) the *Proſtaphæreſis* or Equation of the Moon's Centre, as in the common way: and this being taken from the former Semicircle of the middle Anomaly, and *added* in the latter to the Moon's Place thus thrice equated, will produce the Place of the Moon a fourth time equated.

The greateſt Variation of the Moon (*viz.* that which happens when the Moon is in an Octant with the Sun) is, nearly, reciprocally as the Cube of the Diſtance of the Sun from the Earth. Let that be taken 37′. 25″. when the Sun is *in Perigæo*, and 33′. 40″. when he is *in Apogæo:* And let the Differences of this Variation in the Octants be made reciprocally as the Cubes of the Diſtances of the Sun from the Earth; and ſo let a Table be made of the aforeſaid Variation of the Moon in her *Octants* (or its Logarithms) to every *Tenth*, *Sixth*, or *Fifth* Degree of the mean Anomaly: And for the Variation out of the Octants, make, as Radius to the Sine of the double Diſtance of the Moon from the next Syzygy or Quadrature : : ſo let the aforefound Variation in the Octant be to the Variation congruous to any other Aſpect ; and this *added* to the Moon's Place before found in the firſt and third Quadrant (accounting from the Sun) or *ſubducted* from it in the ſecond and fourth, will give the Moon's Place equated a fifth time.

Again, as Radius to the Sine of the Sum of the Diſtances of the Moon from the Sun, and of her Apogee from the Sun's Apogee (or the Sine of the Exceſs of that Sum above 360°.) : : ſo is 2′. 10″. to a ſixth Equation of the Moon's Place, which muſt be *ſubtracted*, if the aforeſaid Sum or Exceſs be leſs than a Semicircle, but *added*, if it be greater. Let it be made alſo, as Radius to the Sine of the Moon's Diſtance from the Sun : : ſo 2 ′. 20 ″. to a ſeventh Equation: which, when the Moon's Light is encreaſing, *add*, but when decreaſing, *ſubtract*; and the Moon's Place will be equated a ſeventh time, and this is her Place *in her proper Orbit*.

Note here, the Equation thus produced by the mean Quantity 2 ′. 20 ″. is not always of the ſame Magnitude, but is encreaſed and diminiſhed according to the Poſition of the Lunar Apogee. For if the Moon's Apogee be in Conjunction with the Sun's, the aforeſaid Equation is about 54 ″. greater: but when the Apogees are in oppoſition, 'tis about as much leſs; and it librates between its greateſt Quantity 3 ′. 14 ″. and its leaſt 1 ′. 26 ″. And this is when the Lunar Apogee is in Conjunction or Oppoſition with the Sun's: But in the Quadratures the aforeſaid Equation is to be leſſen'd about 50″. or one Minute, when the Apogees of the Sun and Moon are in Conjunction; but if they are in Oppoſition, for want of a ſufficient number of Obſervations, I cannot determine whether it is to be leſſen'd or increas'd. And even as to the Argument or Decrement of the Equation 2′. 20″. above mentioned, I dare determine nothing certain, for the ſame Reaſon, *viz.* the want of Obſervation accurately made.

If the ſixth and ſeventh Equations are augmented or diminiſhed in a reciprocal *Ratio* of the Diſtance of the Moon from the Earth, *i. e.* in a direct *Ratio* of the Moon's Horizontal Parallax; they will become more accurate: And this may readily be done, if Tables are firſt made to each Minute of the ſaid Parallax, and to every ſixth or fifth Degree of the Argumennt of the ſixth Equation for the *Sixth*, as of the Diſtance of the Moon from the Sun, for the *Seventh* Equation.



From the Sun's Place take the mean Motion of the Moon's aſcending Node, equated as above; the Remainder ſhall be the Annual Argument of the Node, whence its ſecond Equation may be computed after the following manner in the preceding Figure:

Let T as before repreſent the Earth, TS a Right Line conjoining the Earth and Sun: Let alſo the Line TACB be drawn to the Place of the Aſcending Node of the Moon, as above equated; and let STA be the Annual Argument of the Node. Take TA from a Scale, and let it be to AB : : as 56 to 3, or as 18 $\frac{2}{3}$ to 1. Then biſſect BA in C, and on C as a Centre, with the Diſtance CA, deſcribe a Circle as AFB, and make the Angle BCF equal to double the Annual Argument of the Node before found: So ſhall the Angle BTF be the ſecond Equation of the Aſcending Node: which muſt be *added* when the Node is paſſing from a Quadrature to a Syzygy with the Sun, and *ſubducted* when the Node moves from a Syzygy towards a Quadrature. By which means the true Place of the Node of the Lunar Orbit will be gained: whence from Tables made after the common way, the *Moon's*

Moon's Latitude, and the Reduction of her Orbit to the Ecliptick, may be computed, ſuppoſing the Inclination of the Moon's Orbit to the Ecliptick to be 4°. 59′. 35″. when the Nodes are in Quadrature with the Sun; and 5°. 17′. 20″. when they are in the Syzygys.

And from the Longitude and Latitude thus found, and the given Obliquity of the Ecliptick 23°. 29′. the Right Aſcenſion and Declination of the Moon will be ſound.

The Horizontal Parallax of the Moon, when ſhe is in the Syzygys at a mean Diſtance from the Earth, I make to be 57′. 30″. and her Horary Motion 33′. 32″. 32‴. and her apparent Diameter 31′. 30″. But in her Quadratures at a mean Diſtance from the Earth, I make the Horizontal Parallax of the Moon to be 56′. 40″. her Horary Motion 32′. 12″. 2‴. and her apparent Diameter 31′.

3″. The Moon in an Octant to the Sun, and at a mean Diſtance, hath her Centre diſtant from the Centre of the Earth about 60 $\frac{2}{9}$. of the Earth's Semi-Diameters.

The Sun's Horizontal Parallax I make to be 10″. and its apparent Diameter at a mean Diſtance from the Earth, I make 32′. 15″.

The Atmoſphere of the Earth, by diſperſing and refracting the Sun's Light, caſts a Shadow as if it were an Opake Body, at leaſt to the height of 40 or 50 Geographical Miles (by a Geographical Mile I mean the ſixtieth Part of a Degree of a great Circle, on the Earth's Surface) This Shadow falling upon the Moon in a Lunar Eclipſe, makes the Earth's Shadow be the *larger or broader*. And to each Mile of the Earth's Atmoſphere is correſpondent a Second in the Moon's Disk, ſo that the Semidiameter of the Earth's Shadow projected upon the Disk of the Moon is to be encreaſed about 50 Seconds: or which is all one, in a Lunar Eclipſe, the Horizontal Parallax of the Moon is to be encreaſed in the Ratio of about 70 to 69.

Thus far the Theory of this Incomparable Mathematician. And if we had many Places of the Moon *accurately obſerved*, eſpecially about her Quadratures, and theſe well compared with her Places at the ſame time calculated according to this Theory; it would then appear whether there yet remain any other ſenſible Equations, which when accounted for, might ſerve to improve and enlarge this Theory.

Dr. *Greg. Aſtr. Elem. Phyſ.* & *Geom. p.* 336.

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